Coherent transport of interacting electrons through a single scatterer

Martin Moško, Pavel Vagner, Andrej Gendiar, Radoslav Németh

Institute of Electrical Engineering, Slovak Academy of Sciences, 84104 Bratislava, Slovakia

Abstract

Using the self-consistent Hartree-Fock method, we calculate the persistent current of weakly-interacting spinless electrons in a one-dimensional ring containing a single δ barrier. We find that the persistent current decays with the system length (L) asymptotically like $I \propto L^{-1-\alpha}$, where $\alpha > 0$ is the power depending only on the electron-electron interaction. We also simulate tunneling of the weakly-interacting one-dimensional electron gas through a single δ barrier in a finite wire biased by contacts. We find that the Landauer conductance decays with the system length asymptotically like $L^{-2\alpha}$. The power laws $L^{-1-\alpha}$ and $L^{-2\alpha}$ have so far been observed only in correlated models. Their existence in the Hartree-Fock model is thus surprising.

Key words: one-dimensional transport, mesoscopic ring, persistent current, electron-electron interaction
PACS: 73.23.-b, 73.61.Ey

Magnetic flux applied through the opening of a mesoscopic conducting ring gives rise to a persistent electron current circulating along the ring [1]. Here we study the persistent current of interacting spinless electrons in a one-dimensional (1D) ring with a single scatterer.

For non-interacting electrons the persistent current (I) depends on the magnetic flux (ϕ) and ring length (L) as [2]

$$I = (ev_F/2L)|\tilde{t}_{k_F}|\sin(\phi'), \tag{1}$$

if $|\tilde{t}_{k_F}| \ll 1$. In eq. (1) $\phi' \equiv 2\pi\phi/\phi_0$, $\phi_0 = h/e$ is the flux quantum, $\tilde{t}_k$ is the electron transmission amplitude through the scatterer, k_F is the Fermi wave vector, and v_F is the Fermi velocity. For a repulsive electron-electron interaction the spinless persistent current was derived in the Luttinger liquid model [2]. For $L \to \infty$

$$I \propto L^{-\alpha-1}\sin(\phi'), \tag{2}$$

where $\alpha > 0$ depends only on the e-e interaction.

In this work we find similar results in the Hartree-Fock model. We consider N interacting 1D electrons with free motion along a circular ring threaded by magnetic flux $\phi = BS = AL$, where S is the area of the ring, B is the magnetic field threading the ring, and A is the magnitude of the vector potential. In the Hartree-Fock model the many-body wave function is the Slater determinant of single-electron wave functions $\psi_k(x)$. These wave functions obey the Hartree-Fock equation

$$\left[\frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial x} + \frac{2\pi}{L}\frac{\phi}{\phi_0}\right)^2 + \gamma\delta(x) + U_H(x) + U_F(k,x)\right]\psi_k(x) = \varepsilon_k\psi_k(x) \tag{3}$$

with cyclic boundary condition $\psi_k(x+L) = \psi_k(x)$, where m is the electron effective mass, x is the electron coordinate along the ring, $\gamma\delta(x)$ is the potential of the scatterer, the Hartree potential is given by

$$U_H(x) = \sum_{k'}\int dx' V(x-x')|\psi_{k'}(x')|^2, \tag{4}$$

the Fock term is written as an effective potential

$$U_F(k,x) = -\frac{1}{\psi_k(x)} \times \sum_{k'}\int dx' V(x-x')\psi_k(x')\psi^*_{k'}(x')\psi_{k'}(x), \tag{5}$$

and $V(x-x')$ is the electron-electron (e-e) interaction.

We solve equation (3) coupled with the potentials (4) and (5) using self-consistent numerical iterations [4]. We obtain numerically the single-particle states $\psi_k(x)$

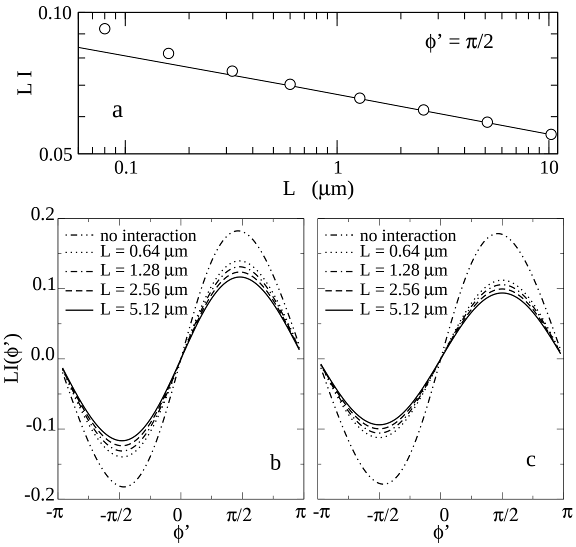

Fig. 1. Panel a: Persistent current $LI(\phi' = \pi/2)$ versus L. The Hartree-Fock data (open circles) follow for large L the scaling law $LI \propto L^{-\alpha}$ shown in a full line, with $\alpha = 0.0855$ obtained as discussed in the text. Panels b and c: Persistent current $LI(\phi')$ for various L; panel b shows the Hartree-Fock results while panel c shows the scaling law of the Luttinger liquid. All data are normalized by the constant $ev_F/2$.

and ε_k, the energy of the Hartree-Fock groundstate, E, and eventually the persistent current $I = -\partial E(\phi)/\partial\phi$.

We present results for the GaAs ring with electron density $n = 5 \times 10^7\ \mathrm{m}^{-1}$, effective mass $m = 0.067\ m_0$, and e-e interaction

$$V(x - x') = V_0\, e^{-\left|x-x'\right|/d}, \tag{6}$$

where $V_0 = 34$ meV and $d = 3$ nm. The interaction (6) is short-ranged. It emulates screening and allows comparison with correlated models [2,3] which also use the e-e interaction of finite range.

We study rings with a strong scatterer ($|\tilde{t}_{k_F}| \ll 1$), for which the asymptotic behavior with L is reachable for not too large L [3]. To show results typical of $|\tilde{t}_{k_F}| \ll 1$, we use the δ barrier with transmission $|\tilde{t}_{k_F}| = 0.03$.

Panel a of figure 1 shows in log scale the persistent current $LI(\phi' = \pi/2)$ as a function of L. The full line is the power law $LI \propto L^{-\alpha}$ predicted by equation (2). For weak e-e interaction ($\alpha \ll 1$) it holds [5] that $\alpha = [V(0) - V(2k_F)]/2\pi\hbar v_F$, where $V(q)$ is the Fourier transform of the e-e interaction $V(x - x')$. The Fourier transform of our interaction (6) reads $V(q) = 2V_0 d/(1+q^2d^2)$. For the above parameters $\alpha = 0.0855$. The full line fits our Hartree-Fock data for large L.

Panel b shows our Hartree-Fock results for $LI(\phi')$. To compare our results with the scaling law (2), we formulate the relation (2) as follows [2]. We replace the bare transmission amplitude $\tilde{t}_{k_F}$ in the non-interacting scaling law (1) by the transmission amplitude of the correlated electron gas, $t_{k_F} \simeq \tilde{t}_{k_F}\,(d/L)^{\alpha}$, which holds

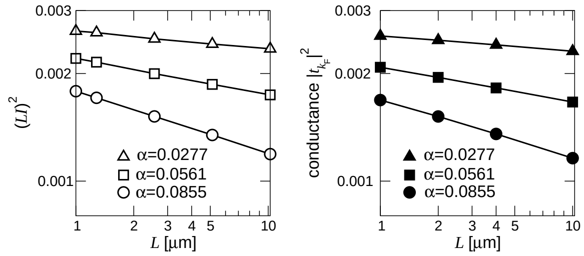

Fig. 2. Left: Square of the persistent current LI as a function of L for $\phi' = \pi/2$ and for various e-e interaction strengths α. Right: Landauer conductance $|t_{k_F}|^2$ of the equivalent 1D wire.

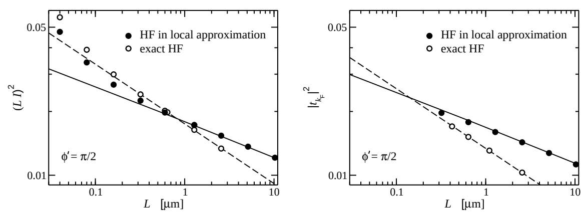

Fig. 3. Left: Square of the persistent current LI as a function of L for $\phi' = \pi/2$ and $\alpha = 0.0855$. Right: Landauer conductance $|t_{k_F}|^2$ of the equivalent 1D wire.

[5] for small $\tilde{t}_{k_F}$. We obtain the scaling law (2) including the proportionality constant $const = ev_F\left|\tilde{t}_{k_F}\right| d^{\alpha}/2$. This scaling law is presented in panel c. It can be seen that the results of panels b and c are in good accord.

Finally, the Hartree-Fock equation (3) can be used to study the conductance of a straight 1D wire biased by contacts, if we omit the term $\propto \phi$. Of course, we also replace the cyclic boundary condition by the boundary conditions of the tunneling problem [5],

$$\psi_k(x = -L/2) = e^{ikx} + r_k e^{-ikx}, \psi_k(x = L/2) = t_k e^{ikx}, \tag{7}$$

where r_k is the reflection amplitude and t_k is the transmission amplitude (analogously for the electrons entering the wire from the right). We have solved this Hartree-Fock problem self-consistently and we have evaluated the Landauer conductance $(e^2/h)|t_{k_F}|^2$.

The result is shown in figure 2 together with the square of LI for the equivalent ring. The conductance scales like $L^{-2\alpha}$ and so does the square of LI.

We thank for the APVT grant APVT-51-021602.

References

[1] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, UK, 2002).

[2] A. Gogolin, N. Prokof'ev, Phys. Rev. B **50**, 4921 (1994).

[3] V. Meden, U. Schollwöck, Phys. Rev. B **67**, 035106 (2003).

[4] R. Németh, M. Moško, cond-mat/0503146; P. Vagner, M. Moško, R. Németh, et al., submitted to Physica E.

[5] K. A. Matveev et al., Phys. Rev. Lett. **71**, 3351 (1993).